4DEnVar with Iterative Calculation of Nonlinear Nonhydrostatic Model Compared to En4DVar

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1. Introduction

En4DVar and 4DEnVar are two popular variational data assimilation methods that use the ensemble-based forecast error covariance. While En4DVar calculates the gradient of the cost function with the adjoint of tangent linear forecast model, 4DEnVar calculates it with the ensemble perturbations. To compare between En4DVar and 4DEnVar, we developed these two methods based on JMA nonhydrostatic model (NHM)-based 4DVar data assimilation system (JNoVA), which iteratively calculates the nonlinear NHM to gain the cost function.

2. Formulation

In JNoVA, to gain the analysis $\mathbf{x}_t = M_t\,(\mathbf{x}_0^f + \mathbf{U}\mathbf{v})$, the cost function

$$J(\mathbf{v}) = \frac{1}{2}\mathbf{v}^T\mathbf{v} + \frac{1}{2}\sum_t [HM_t(\mathbf{x}_0) - \mathbf{y}_t]^T \mathbf{R}_t^{-1} [HM_t(\mathbf{x}_0) - \mathbf{y}_t] \tag{1}$$

is minimized using the gradient of the cost function

$$\nabla J(\mathbf{v}) = \mathbf{v} + \sum_t \mathbf{U}^T \mathbf{M}_t^T \mathbf{H}^T \mathbf{R}_t^{-1} [HM_t(\mathbf{x}_0) - \mathbf{y}_t], \tag{2}$$

where $\mathbf{x}_0^f$ is the first guess, $\mathbf{U}$ is the square root of the climatological background error covariance, M_t and $\mathbf{M}_t$ are the nonlinear forecast model and its tangent linear version, respectively, H and $\mathbf{H}$ are the observation operator and its tangent linear version, respectively, and $\mathbf{y}_t$ and $\mathbf{R}_t$ are the observation and the observation error covariance in time t, respectively. In En4DVar, $\mathbf{U}$ is replaced by initial ensemble perturbations $\mathbf{X}_0=[\delta\mathbf{x}_0^{(1)}\ldots\delta\mathbf{x}_0^{(N)}]/(N-1)^{1/2}$. In 4DEnVar, furthermore, $\mathbf{X}_0^T\mathbf{M}_t^T$ in Eq. (2) is replaced by perturbations of ensemble forecasts $\mathbf{X}_t^T$. Therefore, the nonlocal gradient of the cost function can be reflected in the analysis of 4DEnVar without the adjoint model $\mathbf{M}_t^T$ compared to En4DVar.

3. Data Assimilation Experiments with Lorenz-63 model and JNoVA

To clarify the advantage of 4DEnVar compared to En4DVar, we conducted the data assimilation experiments with the Lorenz-63 model (the number of ensemble members: 3). The result showed that 4DEnVar can further minimize the cost function than En4DVar if the gradient of the cost function is nonlinear and the ensemble perturbations are appropriate (Fig. 1).

In the single observation assimilation experiments with JNoVA, the analysis of 4DEnVar was different from that of En4DVar (Fig. 2). In both analyses, however, the observation was assimilated as expected. The difference between En4DVar and 4DEnVar is probably caused by the nonlinearity of the NHM.

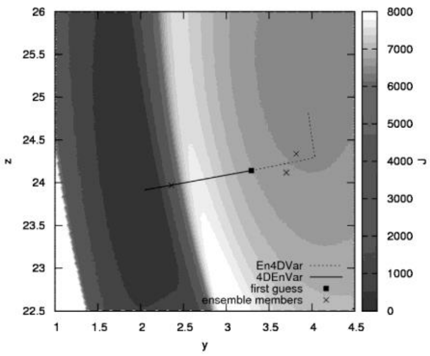

Fig.1. The cost function (shaded) and the trajectory of its minimization by En4DVar (dotted line) and 4DEnVar (solid line) with the Lorenz-63 model. ■ and × are points of the first guess and the three members, respectively.

4. Summary

We developed En4DVar and 4DEnVar based on JNoVA and examined the difference of them. As a next step, we will develop the time evolution of the climatological background error covariance to improve the hybrid of forecast error covariance in 4DEnVar.

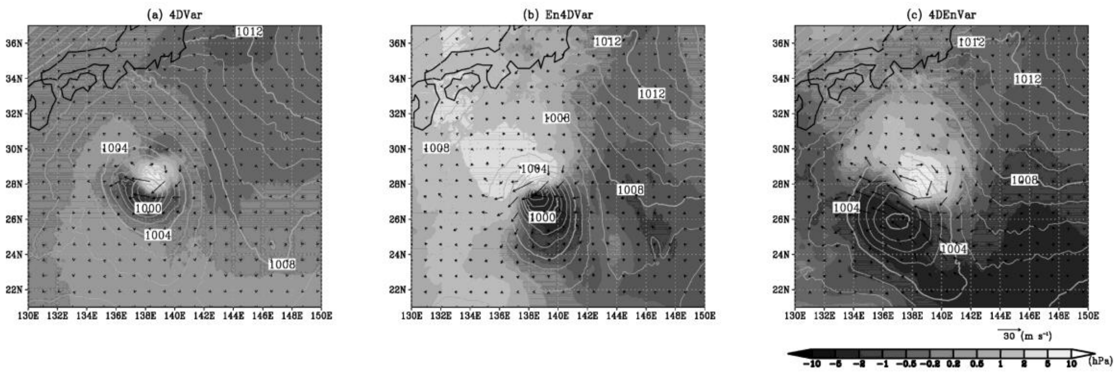

Fig.2. The analysis increments of horizontal wind (arrows, m s^{-1}, 1180-m height) and surface pressure (shaded, hPa) along with the analyzed surface pressure (contours, hPa) at the time of observation in the single observation assimilation experiment with JNoVA (a: 4DVar; b: En4DVar; c: 4DEnVar). In each experiments, zonal wind –30.0 m s^{-1} (innovation: –47.7 m s^{-1}) is assimilated in (138.7E, 27.7N) at 1180-m height at the end of the 3-hour assimilation window and the number of ensemble members is 50.

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